

Kondo phase diagram of quark matter

Shigehiro Yasui,^{1,*} Kei Suzuki,¹ and Kazunori Itakura^{2,3}

¹*Department of Physics, Tokyo Institute of Technology, Tokyo 152-8551, Japan*

²*KEK Theory Center, Institute of Particle and Nuclear Studies,*

High Energy Accelerator Research Organization, 1-1, Oho, Ibaraki, 305-0801, Japan

³*Graduate University for Advanced Studies (SOKENDAI), 1-1 Oho, Tsukuba, Ibaraki 305-0801, Japan*

We discuss the ground state of a quark matter containing heavy quarks as impurities in a simple model which exhibits the QCD Kondo effect. The model includes a current-current interaction with the color exchange between a light quark (ψ) and a heavy quark (Ψ). We introduce a gap function $\Delta \sim \langle \bar{\psi}\Psi \rangle$ which represents the correlation between ψ and Ψ , and perform the mean-field approximation assuming that heavy quarks are uniformly distributed. Values of the gap Δ measure the strength of mixing between ψ and Ψ . The gap equation obtained from the minimum of the thermodynamical potential together with the condition for the heavy-quark number conservation turns out to allow for nonzero values of the gap as the most stable state. We draw a phase diagram in μ (the light-quark chemical potential) and λ (an analog of the heavy-quark chemical potential) plane, and identify the region where the QCD Kondo effect occurs.

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Introduction.—The Kondo effect originally refers to the increasing behavior of the resistance of a metal containing magnetic impurities with decreasing temperature and is now understood as resulting from the enhanced non-Abelian (spin-exchange) interaction between the conducting electron and the impurity as first revealed by J. Kondo [1]. The Kondo effect occurs for any small coupling, and leads to formation of the “Kondo cloud” – a composite state made of conducting electrons and the impurity [2–4]. The concept behind the Kondo effect has been recognized as one of the most important ones in modern physics and can be universally applied to various systems from metals to artificial materials such as quantum dots and quantum devices [5–11]. The essence for the Kondo effect to occur is compactly summarized by the following four ingredients [2–4]: (i) heavy impurities, (ii) Fermi surface (degeneracy) of fermions, (iii) quantum fluctuations (loop effect), and (iv) non-Abelian interaction.

Recent progress includes the addition of new systems to realize the Kondo effect – the nuclear/quark matter governed by the strong interaction [12–15] [16]. At very high densities, a quark matter is formed by light quarks liberated from hadrons and is well approximated by a relativistic free Fermi gas with the typical Fermi energies $\epsilon \sim \mu$ larger than $\Lambda_{\text{QCD}} \sim 200\text{MeV}$ [17–19]. Compared to the typical scale $\mu \gtrsim \Lambda_{\text{QCD}}$ of the quark matter, charm and bottom quarks with masses $m_c \simeq 1.275\text{ GeV}$ and $m_b \simeq 4.660\text{ GeV}$ [20] can be regarded as heavy impurities. In this case, we can confirm that the conditions (i)–(iv) are satisfied, where the non-Abelian interaction between a light quark and a heavy quark is given by an SU(3) color (gluon) exchange. This is called the *QCD Kondo effect*. Since the quark matter experimentally created in high-energy heavy-ion collisions contains not only light (up, down and strange) quarks but also heavy

(charm and bottom) quarks, we expect that the QCD Kondo effect would affect various properties of the quark matter through the changes of thermodynamic properties (e.g. heat capacity, susceptibility) as well as of transport properties (e.g. electric/color-electric conductivity).

Previously, the QCD Kondo effect was studied by the perturbative renormalization group equation obtained at the one-loop level [12–14]. In this Letter, to investigate the ground state of the quark matter with heavy impurities, we develop a non-perturbative analysis. Recall that the Kondo effect is induced by a kind of instabilities around the Fermi surface, similarly to superconductivity and chiral symmetry breaking. Thus it is a first natural step towards a nonperturbative analysis to introduce a gap associated with the instability mode and perform the mean-field approximation. Indeed, such analysis is known to be successful in the investigation of the ground state of the Kondo system in condensed matter physics [21, 22] (see also for the Kondo lattice [23], the quantum dots [24] and Dirac fermions [25, 26]). Recently, the isospin Kondo effect in “charm (bottom) nuclei” was also studied in the mean-field approach by one of the authors [15]. In this work, we will determine the ground state of the quark matter in the presence of the QCD Kondo effect within the mean-field analysis with light and heavy quark densities being varied, and will plot a “phase diagram” for the Kondo effect.

Lagrangian.—We consider a model having the current-current interaction with the color exchange between a light (massless) quark (ψ) and a heavy quark (Ψ) with mass m_Q [27–30]:

$$\mathcal{L} = \bar{\psi}i\not{\partial}\psi + \mu\bar{\psi}\gamma^0\psi + \bar{\Psi}i\not{\partial}\Psi - m_Q\bar{\Psi}\Psi - G_c \sum_a (\bar{\psi}\gamma^\mu T^a \psi) (\bar{\Psi}\gamma_\mu T^a \Psi), \quad (1)$$

where $G_c > 0$ and $T^a = \lambda^a/2$ ($a = 1, \dots, N_c^2 - 1$) are

the generators of $SU(N_c)$ with λ^a being the Gell-Mann matrices and $N_c = 3$ in QCD. The coupling strength G_c is taken to be positive so that the interaction reproduces the attraction in the color anti-triplet channel mimicking the property of the one-gluon-exchange interaction. We consider N_f flavors for light quarks: $\psi = (\psi_1, \dots, \psi_{N_f})^t$. For simplicity, flavor indices are not explicitly written for the fields ψ and we assume that they have the common chemical potential μ . We note that \mathcal{L} is invariant under both the chiral symmetry for light quarks and the heavy-quark spin symmetry for heavy quarks.

We can simplify the Lagrangian (1) in the heavy quark limit [31, 32]: We separate the heavy quark momentum p^μ into the on-mass-shell part $m_Q v^\mu$ and the off-mass-shell part k^μ as $p^\mu = m_Q v^\mu + k^\mu$ with v^μ being the four-velocity of the heavy quark ($v^\mu v_\mu = 1$). The energy scale of k^μ should be much smaller than m_Q . We focus on the dynamics concerning the residual momentum k^μ . Namely, we replace Ψ by the effective field Ψ_v having the momentum k^μ as $\Psi \rightarrow \Psi_v = \frac{1}{2}(1 + \not{v})e^{im_Q v \cdot x}\Psi$. Concerning the four-point interaction, we use the Fierz transformation, $\sum_a (T^a)_{ij}(T^a)_{kl} = (1/2)\delta_{il}\delta_{kj} - (1/2N_c)\delta_{ij}\delta_{kl}$, for the color part, and rewrite the interaction term including the minus sign which appears when we exchange fermion fields. It is important to use the Fierz transformation only for the $SU(N_c)$ generator, not for Dirac matrices. We note that the term proportional to $\bar{\psi}_\alpha \psi_\beta$ can be omitted because it gives no color-flipping and can be neglected in the large N_c limit. Below, we consider the static limit for the impurity motion $v^\mu = (1, \vec{0})$ and use the relation $\bar{\Psi}_v \Psi_v = \Psi_v^\dagger \Psi_v$.

As for the heavy quark number density, we consider the condition, $\Psi_v^\dagger(x)\Psi_v(x) = \sum_i \delta^{(3)}(\vec{x} - \vec{x}_i) \rightarrow n_Q \equiv \overline{\sum_i \delta^{(3)}(\vec{x} - \vec{x}_i)}$, with the three-dimensional delta function $\delta^{(3)}(\vec{x})$ and \vec{x}_i being the position of the impurity i . $\overline{\sum_i}$ indicates average over the whole system. Thus, n_Q is the averaged number density of impurities. When we consider the impurity distribution in space, we tend to suppose that a single heavy quark exists like a single point in space. In contrast, for the present analysis, we suppose that the heavy quarks are distributed *uniformly* in space, and the density is sufficiently large so that the averaged distance between heavy quarks is smaller than the coherence length for the QCD Kondo effect $\simeq 1/|\Delta|$, which will be explained later. Due to the uniformity of the heavy quark distribution, the ground state does not break the translational invariance in contrast to the case with a single heavy quark impurity. In addition, as mentioned before, we assume that all the heavy quarks are static and do not propagate in space.

Mean-field approximation. — It has been known within the perturbative renormalization group analysis [12] that the model (1) exhibits the QCD Kondo effect for a single impurity case. We treat here the same model in a non-perturbative way when the impurities are homoge-

neously distributed. As commented before, we consider the color-singlet correlation between a light quark and a heavy impurity $\langle \bar{\psi}_\alpha \Psi_{v\delta} \rangle$ (and its conjugate) which is associated with the instabilities in the QCD Kondo effect. Then we perform the mean-field approximation in the Lagrangian (1) (represented with respect to the effective heavy impurity field Ψ_v):

$$(\bar{\psi}_\alpha \Psi_{v\delta})(\bar{\Psi}_{v\gamma} \psi_\beta) \rightarrow \langle \bar{\psi}_\alpha \Psi_{v\delta} \rangle \bar{\Psi}_{v\gamma} \psi_\beta + \langle \bar{\Psi}_{v\gamma} \psi_\beta \rangle \bar{\psi}_\alpha \Psi_{v\delta} - \langle \bar{\psi}_\alpha \Psi_{v\delta} \rangle \langle \bar{\Psi}_{v\gamma} \psi_\beta \rangle, \quad (2)$$

where we have neglected the term of the second order with respect to fluctuation. The expectation value, $\langle \bar{\psi}_\alpha \Psi_{v\delta} \rangle$ (or $\langle \bar{\Psi}_{v\gamma} \psi_\beta \rangle$), is evaluated by using the ground state wave function. It is important to note that the mean-fields are the Dirac matrices with indices α and δ (or β and γ).

Physically, the mean field $\langle \bar{\psi}_\alpha \Psi_{v\delta} \rangle$ should be directly related to the formation of a bound state of a light quark “hole” and a heavy quark. This is analogous to the formation of a σ meson in association with the quark-antiquark ($\bar{q}q$) condensate in vacuum [27–30].

Let us define the gap function as

$$\Delta_{\delta\alpha} \equiv \frac{G_c}{2} \langle \bar{\psi}_\alpha \Psi_{v\delta} \rangle, \quad (3)$$

whose Dirac structure is further parametrized in momentum space as $\Delta_{\delta\alpha} = \Delta \left(\frac{1+\gamma_0}{2} (1 - \hat{k} \cdot \vec{\gamma}) \right)_{\delta\alpha}$ with a scalar (complex) parameter Δ and $\hat{k} = \vec{k}/|\vec{k}|$. Notice that Δ is independent of momentum, which reflects the translational invariance of the ground state. Plugging this form of the gap, we finally find the mean-field Lagrangian in the momentum space ($v^\mu = (1, \vec{0})$):

$$\begin{aligned} \mathcal{L}^{\text{MF}} = & \bar{\psi} \not{k} \psi + \mu \bar{\psi} \gamma^0 \psi + \bar{\Psi}_v v \cdot k \Psi_v - \lambda (\Psi_v^\dagger \Psi_v - n_Q) \\ & + \Delta \bar{\Psi}_v \frac{1+\gamma_0}{2} (1 + \hat{k} \cdot \vec{\gamma}) \psi \\ & + \Delta^* \bar{\psi} (1 + \hat{k} \cdot \vec{\gamma}) \frac{1+\gamma_0}{2} \Psi_v - \frac{8}{G_c} |\Delta|^2, \end{aligned} \quad (4)$$

where we have added the term with the Lagrange multiplier λ to include the constraint for the number conservation of the heavy quarks. This constraint is necessary because the heavy quark number density should be conserved on average in the mean field (3) (see Refs. [21, 22, 24]). Note that λ can be regarded as the energy cost for putting a heavy quark into a quark matter similar to the chemical potential. Since the mean-field Lagrangian allows for mixing between the fields ψ and Ψ_v , we diagonalize it by the Bogoliubov-like transformation to find the following energy-momentum dispersion relations for the right-handed light quark

$$E_\pm(k) \equiv \frac{1}{2} \left(k + \lambda - \mu \pm \sqrt{(k - \lambda - \mu)^2 + 8|\Delta|^2} \right), \quad (5)$$

$$\tilde{E}(k) \equiv -k - \mu, \quad (6)$$

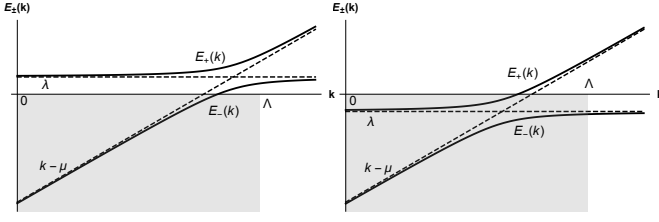


FIG. 1. Dispersion relations of quark $E_\pm(k)$ with finite gap for positive (left) and negative (right) values of λ . The gray band indicates the region of the integrals, $0 < k < \Lambda$.

with $k = |\vec{k}|$. We obtain the same result for the left-handed light quark. The relations (5) and (6) are given by the bases of the linear combination of ψ and Ψ_v . Notice that the mixing takes place only between a positive energy light quark and a heavy quark impurity. The dispersions should possess information about the properties of the ground state in the single-particle picture in the mean-field approximation. In Fig. 1 we show the schematic picture of the dispersions of Eq. (5) for positive (left) and negative (right) values of λ . Because we consider high density states, we neglect the negative-energy component (6). By using Eq. (5), we analyze the thermodynamic potential of the ground state.

It is important to note that the three-dimensional momentum of a light quark, k , is a conserved quantity because the present mean-field maintains the translational invariance of the ground state. This is the case as long as the uniform density distribution of the heavy quark is considered.

In the numerical calculation, we adopt the three-dimensional momentum cutoff because the Lorentz symmetry is violated at finite density. We use the parameter values from the usual Nambu–Jona-Lasinio (NJL) model for $N_f = 2$: $G_c = (9/2)2.0/\Lambda^2$ and $\Lambda = 0.65$ GeV. They are determined by reproducing the quark condensate and the pion decay constant in vacuum [29, 30].

Thermodynamic potential.—Thermodynamic potential computed from the dispersion relations (5) reads

$$\Omega(T, \mu, \lambda; \Delta) = N_f \left[2N_c \int_0^\Lambda f(T, \mu, \lambda; k) \frac{k^2 dk}{2\pi^2} + \frac{8}{G_c} |\Delta|^2 - \lambda n_Q \right], \quad (7)$$

with $f(T, \mu, \lambda; k) = -\beta^{-1} \log(1 + e^{-\beta E_+(k)})(1 + e^{-\beta E_-(k)})$ and $\beta = 1/T$ being the inverse temperature. The factor two in the coefficients of the integral comes from the sum of the right- and left-handed light quarks. We introduce Λ to make the integral finite.

The value of $|\Delta|$ is determined by the minimum of $\Omega(T, \mu, \lambda; \Delta)$ or the solution to the gap equation: $\frac{\partial}{\partial \Delta^*} \Omega(T, \mu, \lambda; \Delta) = 0$. The chemical potential μ and the Lagrange multiplier λ are related to the number densities of light quarks and heavy quarks, n_q and n_Q , respectively: $-\frac{\partial}{\partial \mu} \Omega(T, \mu, \lambda; \Delta) = n_q$ and $\frac{\partial}{\partial \lambda} \Omega(T, \mu, \lambda; \Delta) = 0$.

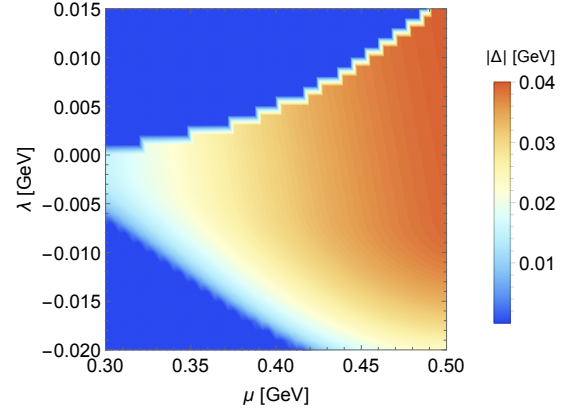


FIG. 2. The gap $|\Delta|$ as a function of μ and λ at $T = 0$ GeV.

The analysis is analogous to that in the color superconductivity with the NJL-type model [19, 33].

Results.— Let us consider the case of zero temperature ($T = 0$). Before presenting numerical results, it is instructive to investigate the approximate analytic solution for the gap at $\lambda \simeq 0$. From Eq. (7), we obtain the gap equation

$$\Delta = N_c G_c \int_0^\Lambda \frac{\Delta}{\sqrt{(k-\mu)^2 + 8|\Delta|^2}} \frac{k^2 dk}{2\pi^2}. \quad (8)$$

This gives two solutions: $|\Delta| = 0$ and

$$|\Delta| \simeq \alpha \sqrt{\frac{\mu(\Lambda - \mu)}{2}} \exp\left(-\frac{2\pi^2}{N_c \mu^2 G_c}\right), \quad (9)$$

where α is a factor independent of G_c and is approximately given by $\alpha = \exp((\Lambda^2 + 2\Lambda\mu - 6\mu^2)/4\mu^2)$ for a small $|\Delta|$. The latter solution gives the most stable state. Importantly, the finite gap always exists for any small coupling constant $G_c > 0$. It is also interesting to notice that the gap contains the exponential factor $\exp(-2\pi^2/(N_c \mu^2 G_c))$ which is common with the factor appearing in the Kondo scale, and thus increases with increasing coupling strength.

The gap equation $\frac{\partial}{\partial \Delta^*} \Omega(T = 0, \mu, \lambda; \Delta) = 0$ for finite values of μ and λ must be solved numerically. The result for the gap $|\Delta|$ as a function of μ and λ is shown in Fig. 2, which essentially corresponds to the phase diagram on the μ - λ plane. We find that $|\Delta|$ increases as μ increases. This is reasonable because the Fermi surface area, and thus the density of states at the Fermi surface, becomes larger with the increasing Fermi momentum. Note also that we have checked that the analytic solution (9) is relatively a good approximation to the numerical result at least for $\lambda \simeq 0$. For example, Eq. (9) gives $|\Delta| = 0.037$ GeV at $\mu = 0.5$ GeV, which is approximately consistent with the full numerical result, $|\Delta| = 0.041$ GeV. The existence of a finite gap indicates that the ground state of the matter is not the normal phase but is the one with

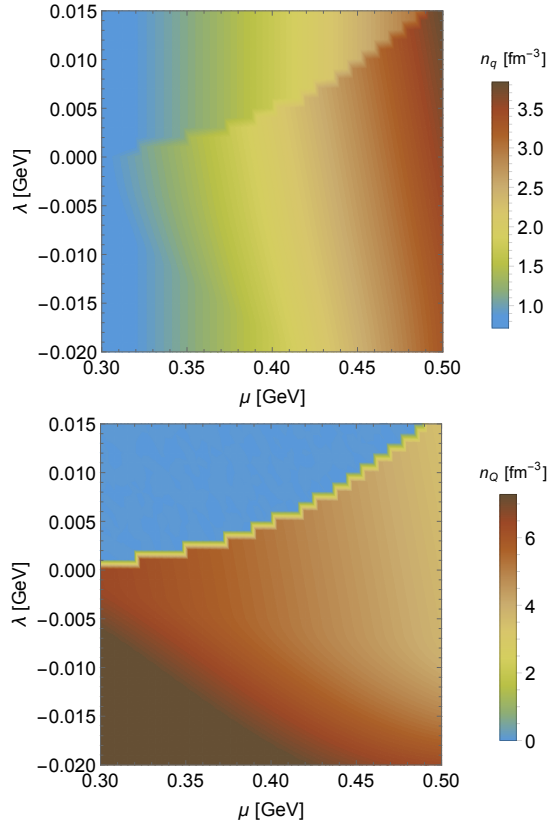


FIG. 3. The number density of light quark n_q and heavy quark n_Q as functions of μ and λ at $T = 0$ GeV.

mixing between a light quark and a heavy quark, which we may call the “Kondo phase”.

In Fig. 3, the number densities of light quarks and heavy quarks, n_q and n_Q , are shown. They are not control parameters but must be dynamically determined through the thermodynamical potential. For $\mu = 0.5$ GeV, we obtain $n_q = 3.8 \text{ fm}^{-3}$ and $n_Q = 1.8 \text{ fm}^{-3}$ at $\lambda = 0.01$ GeV, and $n_q = 3.4 \text{ fm}^{-3}$ and $n_Q = 2.0 \text{ fm}^{-3}$ at $\lambda = -0.01$ GeV.

The thermodynamic potentials $\Omega(T, \mu, \lambda; \Delta)$ at finite temperatures ($T \neq 0$) are plotted as functions of $|\Delta|$ in Fig. 4. Fixing $\mu = 0.5$ GeV and $\lambda \simeq 0$ GeV, we have $|\Delta| = 0.040$ GeV at $T = 0$ GeV. The gap decreases with increasing temperature, e.g. $|\Delta| = 0.034$ GeV at $T = 0.01$ GeV, and it becomes zero at $T = 0.017$ GeV.

Discussions.— The system prefers to form a finite gap when the crossing point $(E, k) = (\lambda, \lambda + \mu)$ of the two original dispersions, $E = k - \mu$, $E = \lambda$, is close to the Fermi surface (see Fig. 1). This is because a finite gap reduces the total energy most effectively when two new dispersions $E_{\pm}(k)$ are asymmetrically involved in the Fermi sea. This expectation is indeed confirmed in the numerical result. Figure 2 shows that when λ becomes positively/negatively large, the gap $|\Delta|$ becomes small.

The results for $|\Delta|$, n_q , and n_Q have a similar structure with a sudden change in the positive λ region (though it

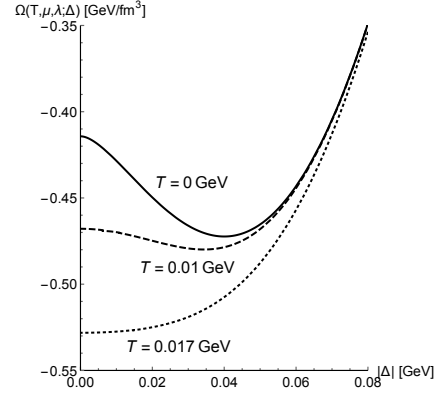


FIG. 4. The thermodynamic potential $\Omega(T, \mu, \lambda; \Delta)$ as a function of $|\Delta|$ with $\mu = 0.5$ GeV and $\lambda \simeq 0$ GeV for several temperatures.

is faint for n_q). Figures 2 and 3 show that the (uniform) gap is formed when the heavy quark density is high. This is reasonable because a uniform condensate will be realized when the correlation/coherence length $\ell \sim 1/|\Delta|$ is much longer than the averaged inter-heavy-quark distance $d \sim 1/n_Q^{1/3}$. When n_Q is small enough, it becomes difficult to maintain a spatially uniform gap. Therefore there is a close correlation between the behaviors of $|\Delta|$ and n_Q .

Summary and outlook.—We discussed the quark matter with heavy quarks as impurity particles. We introduced the color exchange interaction and applied the mean-field approximation to the condensate composed of a light quark and a heavy quark. We found the Kondo phase where the finite condensate is formed in the ground state.

In future, to go beyond the mean-field approximation, we can apply the random-phase approximation (RPA) as studied in Ref. [15]. This is important because the gap is associated with the formation of a bound state, which is described by the fluctuation around the ground state within the RPA. It is also interesting to apply other non-perturbative approaches developed in condensed matter physics, such as the numerical renormalization group [34], the energy-variation, the Green’s function method and the Bethe ansatz (see Refs. [2–4]).

Furthermore, the present analysis does not include the correlations between light quarks, such as the quark-(anti)quark condensate for the color superconductivity and the spontaneous chiral symmetry breaking. Such situations can be studied as an extension of the current approach by using the mean-field theory.

The QCD Kondo effect may be analogous to the Kondo effect for Dirac fermions in condensed matter of electrons [25, 26, 35] [36]. The Kondo effect in Majorana fermions in topological matter is also discussed [37–40]. In addition, it was recently shown that the QCD Kondo effect emerges for the degenerate states in the

Landau level under magnetic field instead of the Fermi surface [14]. The Kondo phase diagram in various environments might be interesting. Those subjects will be left for future works.

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* yasui@th.phys.titech.ac.jp

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